Three-Dimensional FDTD Analysis of Quasi-Optical Arrays Using Floquet Boundary Conditions and Berenger's PML

A. Alexanian, N. J. Kolias, R. C. Compton, and R. A. York

Abstract—**Infinite periodic grid structures excited by normally incident beams are analyzed using finite-difference time-domain (FDTD), with Berenger's PML (perfectly matched layer) absorbing boundary condition used to terminate the computation domain along the beam axis. Floquet boundary conditions are used to handle arbitrarily shaped unit cells. Restriction to normal incidence permits using a Gaussian pulsed excitation to generate the wideband frequency response. The technique is used to model a previously reported multilayer quasioptical rotator array, with excellent agreement to the measurements obtained in the 26.5–40 GHz band in a lens-focused test setup.**

I. Introduction

PLANAR periodic antenna or grid structures are useful in quasioptical power combining schemes for coupling energy between an array of active devices and propagating Gaussian beams. The scattering properties of a finite-sized grid are typically determined by assuming an infinite grid and subsequently reducing the problem to a unit-cell analysis via either some symmetry property of the array or using Floquet analysis. Some recent work [2]–[5] has described application of the finite-difference time-domain (FDTD) method to analysis of such periodic arrays, but has been limited to either two-dimensional (2-D) arrays or monochromatic excitation. There also has been little or no direct validation of the approach with respect to measurements. We have employed the three-dimensional (3-D) FDTD [1] method to determine the transmissive and reflective properties of infinite periodic structures in a manner that is most useful for quasioptical grid designs. We restrict our attention to normally incident beams on arbitrarily shaped unit cells (Fig. 1), which permits a simple frequency-independent Floquet boundary condition, and hence pulsed excitation, for determining the wideband frequency response. This method is amenable to multilayer or cascaded periodic structures, assuming each layer has a common periodicity. Berenger's PML (perfectly matched layer) absorbing boundary condition was used to terminate the mesh along the axis of propagation, which was anticipated to adequately absorb high-order Floquet (waveguide) modes [7]. No instabilities were observed in the absorbing layers for the range of structures and simulation times that were considered in this work. For program validation, the technique was applied to a multilayer quasioptical polarization rotator array, which was previously reported in connection with a beam amplifier project [10]. The geometry analyzed involves two cascaded planar periodic metallizations separated by a dielectric substrate layer. The theoretical results are experimentally validated in the 26.5–40 GHz band using a lens-focused measurement setup.

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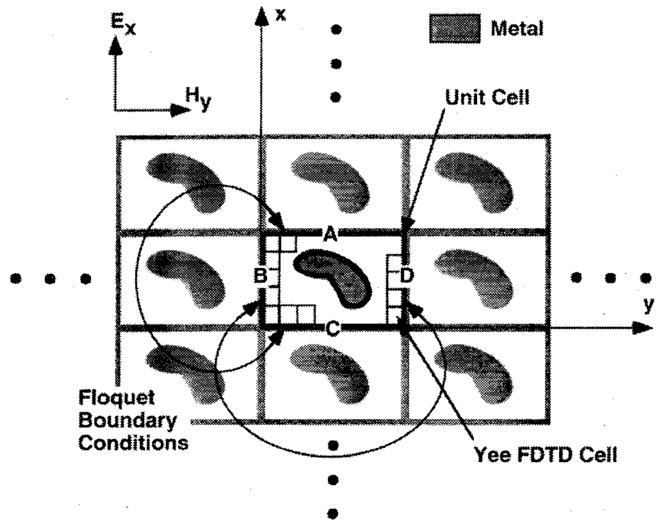

Fig. 1. Top view of infinitely periodic array (in the x and y directions). Excitation consists of a plane wave traveling in the z direction (E_x, H_y). Unit cell and Floquet boundary conditions are also shown.

II. Geometry and Methodology

The grid structure and equivalent unit cell under consideration are shown in Figs. 1 and 2, respectively. The grid structure extends to infinity in the x and y direction, and a linearly polarized plane wave is normally incident on the surface. The unit cell can contain an arbitrary metallization and has sides A,B,C and D (Fig. 1). The periodicity of the problem, coupled with the form of the excitation, dictate that fields on side B have to be identical to those on side D and, similarly, for sides A and C. The mesh effectively wraps around on itself in the x and y directions, which is an extremely simple condition to implement in FDTD. This is obviously just a special case of Floquet's theorem, which has already been applied

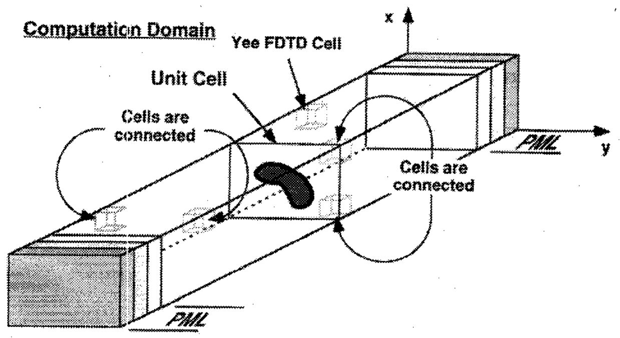

Fig. 2. FDTD mesh. PML regions terminate mesh in the z (longitudinal direction) and Floquet boundary conditions are imposed in the x and y (transverse) directions. Unit cell can be seen inside the mesh.

to FDTD analysis of periodic structures [2]–[4]. Since the incident plane wave impinges normally on the array ($\theta = 0$) the Floquet phase factor ($e^{-jk\sin(\theta)}$, k: wavenumber, θ: angle of incidence) is equal to one. For other angles of incidence, the Floquet factor is complex and explicitly frequency dependent, which typically requires both a monochromatic excitation and some way to deal with complex numbers in the FDTD scheme. The latter has been ingeniously addressed in [2] and [3] using two parallel simulations, one for a cosine (real) and one for a sine (imaginary) excitation. The Floquet phase factor is then incorporated by combining the fields from the two simulations in the appropriate way. The simplified Floquet boundary condition used in this paper is restricted to normal incidence, but it can accommodate a pulsed excitation for faster computation of wideband frequency response. Our approach is most similar to that presented in [4], which was able to deal with arbitrary angles of incidence at the expense of including nearest neighbor cells in the simulation; this places additional demands on CPU time and memory.

The incident plane wave is excited at a plane between the PML region and the grid, which has a Gaussian shape in time. Probes placed before and after the unit cell record the total fields for a reflection and transmission coefficient calculation. The same simulation is also run without the unit cell in place so as to compute the incident field. The data are Fourier transformed to compute the frequency-dependent reflection and transmission characteristics of the structure. To terminate the mesh (Fig. 2) in the z direction we use PML [7], [8] absorbers that end in conducting walls. The Floquet boundary condition is applied to the PML regions in the same manner discussed above. The PML region yielded reflections less than -66 dB over the band of interest for a normally incident TEM pulse and 16 PML layers with quadratically increasing loss. No instabilities were witnessed. This work thus corroborates that of [9], which illustrated the effectiveness of PML in terminating 2-D guided-wave structures (infinite parallel plate guide). There, it was also shown that PML absorbs not only the normally incident fundamental TEM mode, but also higher-order modes, successfully. In most quasioptical arrays of interest, the unit cell is usually chosen smaller than wavelength, which insures that high order modes are evanescent. However, the capacity of the PML region to absorb high order modes efficiently means that smaller computation domains can be used, thus decreasing the computation time.

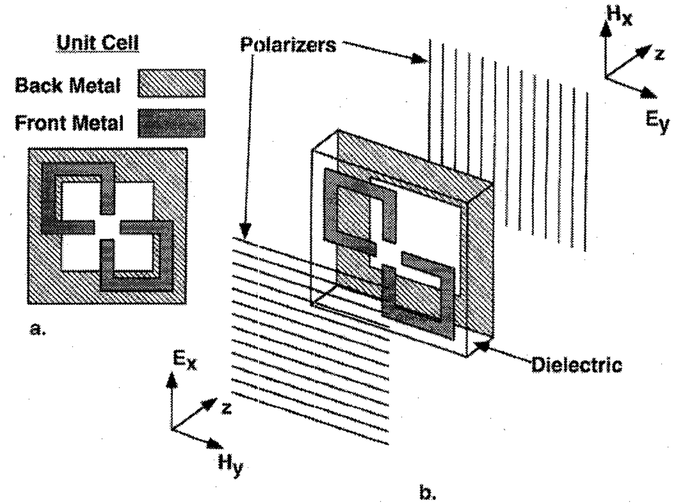

Fig. 3. Quasi-optical rotator array. (a) Top view of unit cell. (b) Perspective view of the unit cell and polarizers.

Qualitatively, the simulation resembles a rectangular waveguide analysis problem. The difference lies in the transverse boundary conditions that are Floquet instead of typical electric or magnetic walls. In [5] electric (TE) or magnetic (TM) walls are used to terminate the unit cell of the array for a 2-D case. Similar boundary conditions in three dimensions have been used for the analysis of planar arrays in the frequency domain using the induced EMF method [6]. In the latter case, the unit cell is placed in a rectangular waveguide comprised of two electric and two magnetic walls. Such boundary conditions cannot be used to analyze the rotator array we have examined in this letter. Electric or magnetic walls imply a reflection symmetry, whereas Floquet walls imply only that every array element is identical.

III. Experimental Validation

A complex grid structure, for which published results are available [10], was chosen for program validation. The unit cell for the structure is shown in Fig. 3(a). It consists of two periodic planar arrays separated by 0.127 mm of dielectric ($\varepsilon_r = 10.8$). The unit cell dimensions are 2.1 mm $\times$ 2.1 mm. The square aperture (1.73 mm $\times$ 1.73 mm) is centered in the back metal. The front metallization consists of 0.056-mm lines that extend 0.66 mm inside the aperture. The part of these lines lying over the back conductor is a 50-Ω microstrip, whereas the one extending into the aperture acts as a probe. The microstrip is about 0.60 mm away from the aperture edge. This structure receives one polarization and transmits the other. Polarizers (0.254-mm-diameter wires spaced 0.900 mm apart) are placed on either side of the array [Fig. 3(b)]; in the simulation these wires were modeled as thin conducting strips, with two strips per unit cell. The front polarizer is positioned 6.5 mm in front of the top metal and the back polarizer 1 mm behind it. The FDTD mesh had $35 \times 35 \times 100(x, y, z)$ cells (absorber not included) . Each PML region used $35 \times 35 \times 16(x, y, z)$ cells. The FDTD cell dimensions were 0.06 mm $\times$ 0.06 mm $\times$ 0.127

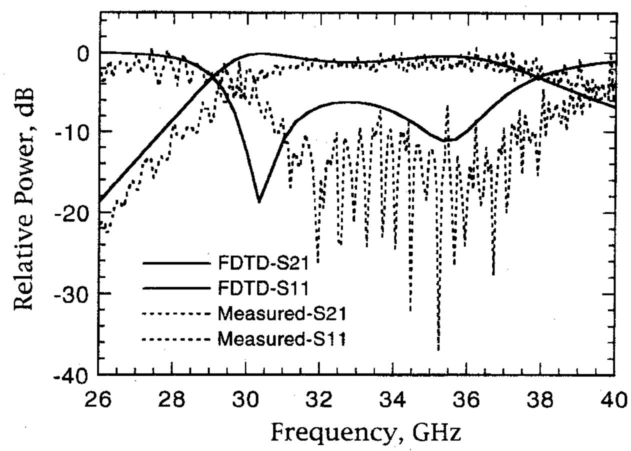

Fig. 4. Comparison between FDTD analysis and measurements for quasioptical rotator array.

mm (x, y, z). The electric conductivity σ_z in the absorber increased from zero to 20 s/m quadratically. This gave us reflections less than -66 dB for a Gaussian TEM pulse whose 35 GHz component dropped to -3 dB (zero dB for dc). The results of the analysis can be seen in (Fig. 4). Agreement is very good considering the complexity of the structure. Experiment and theory both demonstrate a high sensitivity to polarizer positioning. The shift between theoretical and experimental curves is attributed to the difficulty in measuring and simulating the positions of the polarizers accurately.

IV. Conclusion

The 3-D FDTD technique is used to analyze scattering from infinite planar periodic structures excited by normally incident plane waves. Floquet boundary conditions are employed to reduce the problem to the analysis of a unit cell. Transmission and reflection characteristics are determined over a wide band using TEM pulse excitation. Broadband PML absorbers terminate the computation domain in the longitudinal dimension. Finally, agreement between theory and experiment is presented for a quasioptical rotator array.

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